Practice Quiz 2.3: A complicated traffic light

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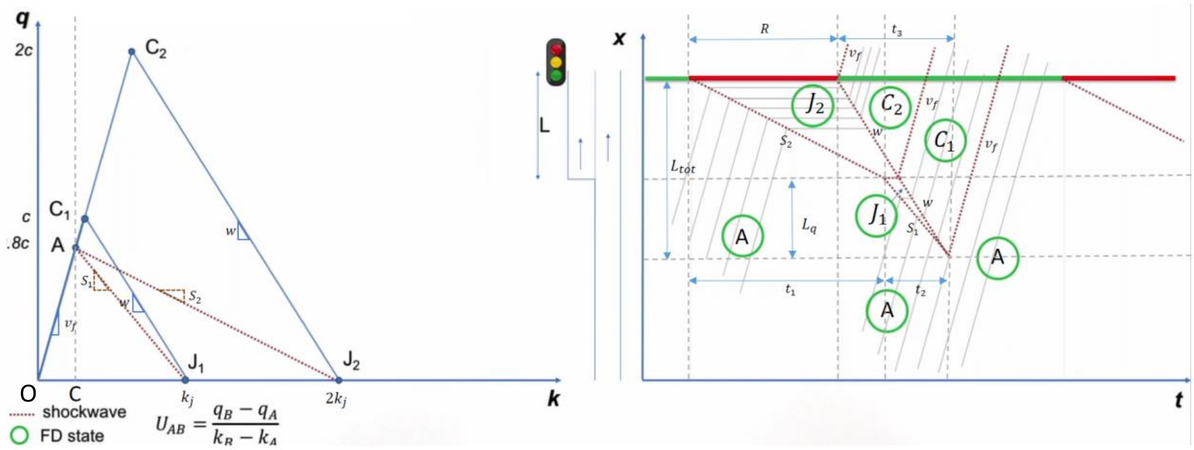
Solution

1. LWR theory is utilized to create the time – space diagram for this problem, as it can be seen below. The states of the system are put on the diagram. The detailed explanation about how to construct of the time-space diagram and the different states observed is provided by Prof. Geroliminis in Video 2.3.



2. We wish to calculate the maximum distance, starting from the position of traffic light, that the queue will reach as it propagates backwards during the red phase, assuming that the arriving flow is always constant and equal to $0.8c$ and length L takes a value that will result in the time-space diagram show above. In other words, we seek to find the length L_{tot}

   Based on the geometry of the time-space diagram we can derive the following equations:

   $S_2 = \frac{L}{t_1} \Leftrightarrow t_1 = \frac{L}{S_2}$ (1)

   $S_1 = \frac{L_q}{t_2} \Leftrightarrow t_2 = \frac{L_q}{S_1}$ (2)

   $w = \frac{L_{tot}}{t_3} = \frac{L+L_q}{t_3} \Leftrightarrow t_3 = \frac{L+L_q}{w}$ (3)

   In addition, we can see that:

$$t_1 + t_2 = R + t_3$$

   By replacing with equations (1), (2) and (3) the last equation becomes:

$$\frac{L}{S_2} + \frac{L_q}{S_1} = R + \frac{L + L_q}{w}$$

By solving for L_q we get:

$$L_q = \frac{S_1\left(L\,w - S_2\,(R\,w + L)\right)}{S_2\,(S_1 - w)} \qquad (4)$$

Based on the geometry of the FD, we have:

$$S_1 = \frac{AC}{CJ_1} = \frac{0.8c}{k_j - (CO)} = \frac{0.8c}{k_j - \frac{0.8c}{v_f}} = \frac{0.8c\,v_f}{k_j v_f - 0.8c},$$

$$S_2 = \frac{AC}{CJ_2} = \frac{0.8c}{2k_j - (CO)} = \frac{0.8c}{2k_j - \frac{0.8c}{v_f}} = \frac{0.8c\,v_f}{2k_j v_f - 0.8c}$$

$$v_f = \frac{0.8c}{k_j - CJ_1} \Leftrightarrow CJ_1 = k_j - \frac{v_f}{0.8c}$$

By substituting S_1 and S_2 in Eq. (4) we can find L_q:

$$L_q = \frac{(2k_j v_f - 0.8c)\left(L\,w - \dfrac{0.8c\,v_f}{2k_j v_f - 0.8c}\,(R\,w + L)\right)}{(k_j v_f - 0.8c)\left(\dfrac{0.8c\,v_f}{k_j v_f - 0.8c} - w\right)} =$$

The maximum distance from the traffic light that the queue will propagate backwards is:

$$L_{tot} = L + L_q$$